Gravity and dual gravity: proposals for an inhomogeneous expanding universe

Jacques Fleuret

Jacques.fleuret@telecom-paristech.org

Key words: gravity; expansion, negative mass; Einstein's equation; cosmological constant; inhomogeneous universe; Hubble

Abstract

I proposed a new expansion force to explain the flat rotation curves of plane galaxies, in a Newtonian approach in 2016. This force is validated here in General Relativity for an inhomogeneous radially symmetrical universe. It can be deduced from the solution of Einstein's equation with a cosmological constant. A pseudo-Schwarzschild "twin-potential" metric is considered, with two different potentials for the space and time dimensions. The expansion force results as a repulsive dual gravity. Negative masses are needed. The necessary mass repartitions are computed, in connection with the radial dynamics and the expansion rate's evolution. A linear model and a quadratic model are studied, connected with the observation representations of a universe in accelerated expansion. It is a first step towards a better representation of an inhomogeneous universe. The framework can be applied to deduce the universe mass repartition form the measured values of the expansion rate variation.

Introduction

The standard model has given a good modelisation of a wide variety of observed phenomena in cosmology. But on one side, exotic dark matter and dark energy are needed to represent a huge part of the total mass and energy. On the other side, they have not been given a real material representation after forty years of intensive research. Furthermore, diverse observed phenomena do pose unsolved questions. As is well known, the flat rotation curves of planar galaxies have been a first challenge, at the origin of the dark matter hypothesis and the MOND theory (Milgrom, 1983). The discovery of expansion acceleration (Riess, 1998; Perlmutter et al., 1997) has been another challenge, at the origin of the so-called dark energy. More recently, the measurement discrepancies of the Hubble Constant have not yet found a satisfactory explanation (Verde & al., 2019) since various estimations of this « constant » give disconnected values. Clearly, estimates for the early universe are about 10% less than those from more recent observations using different standard candles: cepheids, red giants, etc. (Planck, 2018 ; Riess 1 al., 2019 ; Wong, 2019 ; Friedmann & al., 2019 ; Friedmann &al ., 2020).

In a preceding paper, I introduced a new expansion force to explain the flat rotation curves of plane galaxies (Fleuret, 2014). I have shown that this force (proportional to velocity) is able to guarantee a constant transverse velocity. Its radial part is

$$\frac{\dot{r}^2}{r} = H^2 r \quad (1)$$

where $\dot{r}$ and H represent the radial velocity and the expansion rate at the distance r of observation.

This hypothesis has been developed in the context of Newton's theory. I have applied this idea to cosmology, formally obtaining similar results to the FRW's ones, without explicit introduction of neither dark matter nor dark energy. I also proposed a new mass erosion process which could contribute to expansion (Fleuret, 2015). Later on, a Chinese lab studied a very similar force, based on a lagrangian approach, leading to quite similar results (J. Hu and Y. Liu, 2019).

In a most recent paper, I applied this new force to a radially (inhomogeneous) symmetrical space (still under a Newtonian approach), obtaining the necessary mass repartitions for such a model, in conformity with present observations and I showed that negative matter was needed for such a model (Fleuret, 2019).

The homogeneity hypothesis has certainly given relevant simplification to address the difficult problem of the cosmic evolution. But it is not at all certain that our universe is homogeneous. In reality, at different scales, anisotropies have been proved by several observers (YA Baurov & al., 2015; M. R. Wilczynska & al, 2015; Smoot, 1992; Colin 1 al., 2019; S. Konstantinov, 2020; Migkas K & al., 2020): the homogeneity hypothesis is requestioned today.

In this paper, I examine the case of a radially symmetric inhomogeneous space and apply to it the **Einstein's equation of general relativity with a Cosmological Constant Λ**. It will be shown that a new solution can be developed, with **two different potentials for the time and space dimensions**, the **expansion force being nothing else than a gravitational consequence of the relativistic formulation.**

Then the geodesic equation will be solved, for a universe in accelerated expansion. The necessary mass repartition will be obtained, in relation with the radial dynamics and expansion rate variations.

1. The pseudo Schwarzschild twin-potential metric and Einstein's equation

1.1 Metric and Einstein's equation

Let us consider a spherically symmetric space, with the following metric:

$$ds^2 = B(r) - A(r)\, dt^2 - r^2\, dr^2 - r^2\, d\theta^2 - r^2\, sin^2\theta\, d\varphi^2 \qquad (2)$$

With $B(r) = 1 + 2\,\psi$ (3)

And $A(r) = \frac{1}{1+2\,\phi}$ (4)

The two potentials ψ and ϕ are supposed to depend on r only.

The Einstein Equation is then written with a Cosmological Constant $\wedge$.

$$R_{tt} = -\frac{B''}{2A} + \frac{B'}{4A}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{B'}{rA} = -[\wedge + 4\pi G(\rho + 3p)]B \qquad (5)$$

$$R_{rr} = \frac{B''}{2B} - \frac{B'}{4B}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{A'}{rA} = [\wedge - 4\pi G(\rho - p)]A \qquad (6)$$

$$R_{\theta\theta} = -1 + \frac{r}{2A}\left(-\frac{A'}{A} + \frac{B'}{B}\right) + \frac{1}{A} = [\wedge - 4\pi G(\rho - p)]r^2 \qquad (7)$$

To simplify, I suppose that matter is non relativistic, with negligible pressure.

As is well known, the following combination allows to determine A (or ϕ) alone:

$$r^2\left(\frac{R_{tt}}{2B} + \frac{R_{rr}}{2A} + \frac{R_{\theta\theta}}{r^2}\right) = \left(\frac{r}{A}\right)' - 1 = (\wedge - 8\pi G\rho)r^2 \qquad (8)$$

Or, from (4):

$$2(\phi + r\,\phi') = (\wedge - 8\pi G\rho)r^2 \qquad (9)$$

We consider here a (inhomogeneous) spherically symmetric universe, with the following mass repartition:

$$M(r) = \int 4\,\pi r^2\, \rho(r) dr \qquad (10)$$

Of course, this modelisation does not reproduce the probable complexity of the real world. But it presents the advantage of symmetry and it is a first step towards more representative models. A priori, it should be increasing less than r^3, based on several arguments: the Olbers paradigm (W. Olbers, 1823; S.L. Jaki, 1969), stochastic and fractal arguments (B. Mandelbrot, 1977) and the observation of large scale structures with large "empty" zones surrounded by filaments of matter (T. Piran, 1997; A. Benoit-Levy & G. Chardin, 2012).

Another physical question arises about the significance of r= 0. It can be thought of as our present position in the universe. A priori, this position has nothing particular and a change of the central coordinate reference can be envisioned, by a finite translation. To ensure that the long range view of the universe observed from this new reference location should have the same form, it is needed to suppose a spatially infinite universe, or, at least, that the so-called "maximum observed radius R" be much larger than the finite translations allowed for real observers. This is largely realized at the present time…
In this sense, this model envisions a universe spatially expanding to quasi-infinity, in which "any" precise observation point can be considered as a "center".

Eq. (9) can be easily solved, giving $\phi(r)$, in usual units:

$$\phi = \frac{\wedge r^2}{6} - \frac{MG}{c^2 r} \qquad (11)$$

$$\phi' = \frac{\Lambda r}{3} - \left(\frac{MG}{c^2 r}\right)' = \frac{\Lambda r}{3} + \frac{MG}{c^2 r^2} - \frac{M'G}{c^2 r} \qquad (12)$$

Furthermore, eq. (7) results in:

$$\frac{\psi'}{AB} = -\phi' - \frac{2\phi}{r} + \Lambda r - \frac{M'G}{c^2 r} \qquad (13)$$

In the following, ψ and ϕ are supposed to be small.
This equation can be solved for ψ', using (10), (11) and (12):

$$\psi' = \frac{\Lambda r}{3} + \frac{MG}{c^2 r^2} \qquad (14)$$

$$\psi = \frac{\Lambda r^2}{6} + \int \frac{MG}{c^2 r^2} dr \qquad (15)$$

1.2. The geodesic equation

Let us now write the geodesic equation in r (reduced to planar trajectories ($\varphi' = 0$)):

$$\frac{d^2 r}{ds^2} = -c^2 \psi' \left(\frac{dt}{ds}\right)^2 + \phi' \left(\frac{dr}{ds}\right)^2 + r \left(\frac{d\theta}{ds}\right)^2 \qquad (16)$$

(The Christoffel symbols are given in appendix)
This equation can be rewritten in terms of t, considering:

$$\frac{d}{ds}\left(\frac{dr}{ds}\right) = \frac{dt}{ds}\frac{d}{dt}\left(\dot{r}\frac{dt}{ds}\right) = \ddot{r}\left(\frac{dt}{ds}\right)^2 + \frac{\dot{r}}{2}\frac{d}{dt}\left(\frac{dt}{ds}\right)^2 \qquad (17)$$

and $\left(\frac{dr}{ds}\right)^2 = \dot{r}^2 \left(\frac{dt}{ds}\right)^2$ (18)

with (from(2)):

$$\frac{dt}{ds} = \frac{1}{\sqrt{Bc^2 - A\dot{r}^2 - v^2}} \qquad (19)$$

where v is the lateral velocity.
In the following, purely radial trajectories will be considered ($v = 0$).
Eq. (16) becomes:

$$\ddot{r} = \frac{\dot{r}}{2}\frac{(Bc^2 - A\dot{r}^2)^{\cdot}}{Bc^2 - A\dot{r}^2} - c^2\psi' + \dot{r}^2\phi' + \frac{v^2}{r} \qquad (20)$$

From (3) and (4) the additional term can be written (for small ϕ and ψ) as:

$$\frac{1}{c^2 - \dot{r}^2}\left(+\dot{r}^4\phi' - \dot{r}^2\ddot{r} + \dot{r}^2 c^2 \psi'\right) \qquad (21)$$

which, after some rearrangement, leads to:

$$\ddot{r} = -c^2\psi' + \dot{r}^2(\phi' + 2\psi') \qquad (22)$$

According to this important result, the radial acceleration reveals two parts:

- **The first term ($-c^2\psi'$) includes the Newton's force, plus a modification due to the Cosmological Constant.** (This recalls the MOND Theory, except that MOND did not propose the same modification procedure).
- **The second term, proportional to $\dot{r}^2$, and depending on the "space" potential ϕ, represents the expansion force: it is nothing else than the "positive" or "repulsive" part of the gravific acceleration provided by the Einstein's Equation with a Cosmological constant** (in addition to the "negative" classical attraction force (it is not excluded that, for particular physical systems in some circumstances, it could be negative and then possibly represent a contraction).

This result tends to legitimate the first intuitive proposal of such a force, made in the Newtonian context to guarantee constant lateral velocities.

1.3. Resolution of the geodesic equation

Eq. (22) is a differential equation which can be solved to get the radial dynamics, knowing the mass repartition. Let-us suppose that $\dot{r}^2$and $\ddot{r}$ depend on r only:

$$\frac{\dot{r}^2}{c^2} = f(r) \qquad (23)$$

The geodesic equation becomes:

$$f' - 2f(\phi' + 2\psi') = -2\psi' \qquad (24)$$

whose general solution is:

$$f = \frac{\dot{r}^2}{c^2} = -2e^{2(\phi+2\psi)} \int e^{-2(\phi+2\psi)}\, d\psi \qquad (25)$$

Inversely, mass repartition can be deduced from the knowledge of $\dot{r}^2$ or, equivalently the knowledge of the expansion rate $H(r) = \dot{r}/r$. This proposal will be illustrated by some examples in the following, in particular for the universe at large scale. The research of a solution will be guided, in both cases, by what we know of observed data. For this purpose, let-us first review the present observations of the acceleration of the universe.

2. The observed accelerated expansion of the universe

The present observations of the acceleration of the universe expansion can be expressed with two parameters:

-the Hubble Constant Ho

- An acceleration parameter q, which is known to be close to ½.

Then, the radius evolution should have the form (Y-L. Bolotin & al., 2015):

$$r = R\left(1 + H_0 t + q\frac{{H_0}^2 t^2}{2}\right) \qquad (26)$$

$$\dot{r} = RH_0(1 + qH_0 t) \qquad (27)$$

$$\ddot{r} = Rq{H_0}^2 \qquad (28)$$

Where R is supposed to represent the maximum radius of the universe.

Furthermore, we suppose that $\dot{r} = c$ for r=R at time t=0, such as:

$$RH_0 = c \qquad (29)$$

As was done in (Fleuret, 2019), t is eliminated from (26) and (27), leading to the following function $\dot{r}^2(r)$ that should be satisfied to describe present observations:

$$\dot{r}^2 = c^2\left[1 - 2q + 2q\frac{r}{R}\right] \qquad (30)$$

It is a linear expression. Furthermore, since q has been observed to be close to ½, eq. (30) becomes purely linear (incidentally, this is the only function to guarantee a fixed acceleration number $q = \frac{r\ddot{r}}{\dot{r}^2}$):

$$\dot{r}^2 = \frac{c^2}{R} r \qquad (31)$$

And the expansion rate is:

$$H^2 = \frac{\dot{r}^2}{r^2} = \frac{c^2}{Rr} = {H_0}^2\frac{R}{r} \qquad (32)$$

It must be noted that **the expansion rate is not constant** and is inversely proportional to the square root of r in this case.

Let-us now return to GR and solve the geodesic equation for a non-homogeneous universe, to find out whether it gives results in accordance to observations and close to the Newtonian analysis.

3. Mass repartition and negative matter

In the following, I suppose that $\dot{r}^2$and MG are functions of r only and consider the following question: being given $\dot{r}(r)$, what is the needed mass repartition to get it? Let-us add another requirement in order to match the Newtonian approach: is it possible that the right hand-side of eq. (22) could represent the (modified) Newton's force by its first term and the expansion force $\frac{\dot{r}^2}{r}$ by its second term?

This should require:

$$\phi' + 2\psi' = \frac{1}{r} \qquad (33)$$

Using (12) and (14) for ϕ' and ψ' as functions of $\left(\frac{MG}{c^2 r}\right)$, the differential equation can be solved to get the following mass repartition:

$$\frac{MG}{c^2 r} = \frac{1}{2} - \frac{r^2}{{r_0}^2} + \wedge\, r^2 Ln\left(\frac{r}{R}\right) \qquad (34)$$

where r_0 is a constant.

It is notable that **this result implies negative masses (such as predicted by the Newtonian analysis (**Fleuret, 2019**)**. This is not so surprising since, in essence, negative masses induce repulsive forces on positive ones.

Hopefully, there is no doubt on the existence of negative masses, notably with anti-particles. Negative masses have been pointed out to contribute to the development of large scale structures of the universe (T. Piran, 1997), typically arguing that the « empty zones » could be the place where negative masses push away positive ones. They have been introduced into relativistic models in the context of the Dirac-Milne universe (Benoit-Levy & Chardin, 2012; Chardin & Manfredi, 2018; Manfredi & al., 2018) and also into the Schwarzschild metric (Belletête, 2013; Bondarenko, 2019).

The negative mass assumption appears to be an appealing idea to explain expansion by a kind of anti-gravity process (Ni, 2003; Konstantinov, 2020), even if it remains an open subject of research to clarify the way they interact with other masses (Petit, 2014; G. Chardin & G. Manfredi, 2018). Here, we assume the simple idea that negative masses generate repulsive forces towards positive masses and that positive and negative masses are sufficiently separated to avoid more complex interactions (Piran, 1997; Benoit-Levy & Chardin, 2012).

Another question arises about the singularity for Newton's acceleration at $r = 0$. At that point, ψ' is diverging, and our "small potential" hypothesis is no more valid. So, instead of considering that our development describes the universe "seen from a black hole", we better, in this first approach, consider that the central point has no particularity and that our developments are only valid for large radii, and in particular for $r \simeq R$.

So, let-us consider the "far-away" observation $r \simeq R$.

Since it is also the "very old" situation back in time and since at that time, negative masse were supposed to balance positive ones, let-us assume that:

$$\left[\frac{MG}{c^2 r}\right]_{r=R} = 0 \qquad (35)$$

This is another way to admit that positive masses and negative ones are equally represented in the universe.

This allows to determine the constant of integration r_0. It results in the following necessary mass repartition:

$$\frac{MG}{c^2} = \frac{r}{2} - \frac{r^3}{2R^2} + \wedge\, r^3 Ln\left(\frac{r}{R}\right) \qquad (36)$$

It is notable to remark that even if a positive value had been chosen for $M(R)$ as is usually made, negative masses would have been obtained in the resulting mass repartition, due to the $\wedge\, Ln$

term in eq. 36: **negative masses are needed for the twin-potential metric with cosmological constant.**

4. Radial dynamics and expansion rate

Now, from (33), the geodesic equation itself can be rewritten as:

$$\ddot{r} = -c^2\frac{\wedge \mathrm{r}}{3} - \frac{MG}{r^2} + \frac{\dot{r}^2}{r} \qquad (37)$$

And solved easily to obtain the radial dynamics, expressed in terms of velocity or expansion rate:

$$\dot{r}^2 = \frac{c^2}{2}\left(1 + \frac{r^2}{R^2}\right) - c^2 \wedge r^2 Ln^2\left(\frac{r}{R}\right) + c^2\frac{r^2}{R^2}\left(1 - \frac{2\wedge R^2}{3}\right) Ln\left(\frac{r}{R}\right) \qquad (38)$$

$$H^2 = \frac{\dot{r}^2}{r^2} = \frac{c^2}{2r^2}\left(1 + \frac{r^2}{R^2}\right) - c^2 \wedge Ln^2\left(\frac{r}{R}\right) + {H_0}^2\left(1 - \frac{2\wedge R^2}{3}\right) Ln\left(\frac{r}{R}\right) \qquad (39)$$

(the constant of integration has been determined through the requirement that $\dot{r}(R) = c$).

The density can also be deduced from (36) by derivation:

$$\rho = \frac{\rho_c}{3}\left[\frac{R^2}{r^2} + 2\left(\wedge R^2 - \frac{3}{2}\right) + 6 \wedge R^2 Ln\left(\frac{r}{R}\right)\right] \qquad (40)$$

where ρ_c is the critical density: $\rho_c = \frac{3{H_0}^2}{8\pi G}$ (41)

The obtained density has three terms:

- The r^{-2} term which is compatible with the Olbers' question.
- The constant term, which involves two particular values for the cosmological constant :
  It is null for $\wedge R^2 = \frac{3}{2}$
  and it equals ρ_c for $\wedge R^2 = 3$.

  These two cases will be met again later on.

- The last term, proportional to $\wedge$, represents negative masses.
  In order to get a more precise description of the "far-away" situation, let-us now examine what happens in the vicinity of R, where:

$$r = R(1-\epsilon) \qquad \epsilon \ll 1 \qquad (42)$$

4.1. Far away dynamics

For this purpose, eq. (38) is 1st order developed in terms of ϵ, obtaining:

$$\frac{\dot{r}^2}{c^2} = 2\left(1 - \frac{\wedge R^2}{3}\right) - \epsilon\left(3 - \frac{4\wedge R^2}{3}\right) \qquad (43)$$

Rewriting it in terms of r thru (42), we get:

$\frac{\dot{r}^2}{c^2} = \frac{2\wedge R^2}{3} - 1 + \frac{r}{R}\left(3 - \frac{4\wedge R^2}{3}\right)$ (44)

This has the form of eq. (30). To match observations, we make it purely linear under the following condition:

$\wedge R^2 = \frac{3}{2}$ (45)

and the final result is in total conformity with the present observations:

$\dot{r}^2 = \frac{c^2}{R} r$ (31)

Consequently, it has been shown that **our solution of Einstein's equation with the cosmological constant $\frac{3}{2R^2}$ does fit present linear observations.**

What are the implied forces in the process? They are given by the geodesic equation. Near R, **the expansion force is obviously close to $\frac{c^2}{R}$**. And it can be easily checked (by development of (37)) that **the Newton's contribution is $-\frac{c^2}{2R}$**, their difference giving the expansion acceleration.

4.2. Second order resolution and expansion rate variation

Even if eq. (31) can be considered as the solution in conformity with the observed development (26), it can be physically questionable. Such a liner growth with r should imply that $\dot{r}$ could overpass c for r>R, with a constant acceleration suddenly stopped... More generally, as was notably stated by Konstantinov (Konstantinov, 2020):

"Treating the universe as a dynamical system, it is natural to assume that it is non-linear: indeed, linearity is nothing more than an approximation, while non-linearity represents the generic case."

A more physical approach could be to envision a nonlinear curve $\dot{r}^2(r)$, arguing that **our linear observations (eq. (26) to (28)) just represent the tangent of that curve**. Incidentally, according to this approach, the acceleration factor is not restricted to its ½ value, and the expansion rate will be allowed more complex variations than (32), to be obviously checked in regard with the observation measurement results.

In this perspective, let us push the preceding development to the second order in ϵ, and rewrite it in terms of r as above. We now require that $\dot{r}(r) = c$ and $\left[\frac{d}{dr}(\dot{r}^2)\right]_{r=R} = 0$. This now implies:

$\wedge R^2 = 3$ (46)

(in favor of this choice is the fact that the constant part of the density is equal to ρ_c.)

It results in the following dynamics:

$\dot{r}^2 = c^2\left[1 - 4\frac{(r-R)^2}{R^2}\right]$ (47)

Obviously, a non-linear (quadratic) model such as (47) gives more complex variations of $H(r)$, when compared to (32).

Near R, **the expansion force is still $\frac{c^2}{R}$ but now, the Newton's contribution is just the opposite**. These two forces balance together, leading to the fixed radial velocity $\dot{r} = c$.

The two above developments are only possible examples of the relationship between expansion dynamics and mass repartition. In practice**, I suggest that observational precise measurements of $H(r)$ could be the input to eq. (22), allowing to deduce mass repartition**.

Conclusion

I have shown that the expansion force - introduced in the Newtonian analysis of plane galactic rotation curves - results from the Einstein's Equation for a pseudo-Schwarzschild metric with twin potentials. The expansion force is not a 5^{th}. force, neither an "anti-gravity" force. It happens to be a form of dual-gravity.

This study is a first approach towards more precise representation of an inhomogeneous universe, in which negative masses play an important role. A lot of research will be needed to develop more precise modelisations of our complex (probably inhomogeneous) universe and obtain a relevant connection between the mass repartition and the expansion rate.

The debate on the Hubble "constant" is not closed. These results emphasize the fact that very careful analyses are presently required in the process of measurement estimations, to take care of the "standard candles" on one side, and on the presupposed theoretical modelisation underneath on the other side. In principle, if the case of a symmetrical inhomogeneous universe is envisioned, **the experimental results for $H(r)$ could orientate what should be – in accordance with our expressions – the mass repartition of this universe**.

Appendix: Christophel Symbols for the metric (eq. (2))

$$\Gamma^t_{tr} = \frac{B'}{2B} = \frac{\psi'}{1+2\psi}$$

$$\Gamma^r_{tt} = \frac{B'}{2A} = c^2\psi'(1+2\,\phi) \qquad \Gamma^r_{rr} = \frac{A'}{2A} = \frac{-\phi'}{1+2\phi} \qquad \Gamma^r_{\theta\theta} = \frac{-r}{A} = -r(1+2\,\phi)$$

$$\Gamma^\theta_{r\theta} = \frac{1}{r}$$

Acknowledgments

From the very beginning of my research in this field, my proposals have been welcomed by a very small number of open-minded scientists. I am consequently even more grateful to them: Jean Audouze, Jean-Marc Bonnet-Bidaud, Dimitris M. Christodoulou, Denis Gialis, Demos Kasanas, Johan Maszreliez, Michel Mizony, Laurent Nottale, Patrick Petitjean, Robert Schmidt, Jason A. Taylor. I also thank Philippe Maulion and Stephan Mihajlovic of the *Société d'Astronomie Française* for the informations they gave me about respectively recent astrophysics publications and mathematics for cosmology.

References

Milgrom, M., 1983, Astrophys. J, 270, 365

Riess, A.G., C.R. 1998, Astron. J., 116, 1009
Perlmutter et al., 1997, Astrophys. J., 483, 565
Verde et al., arXiv:1907.10625v1 [astro-ph.CO] 24 July 2019

Planck 2018 results, arXiv:1807.06209 [astro-ph.CO]

Adam G. Riess et al., The Astrophysical Journal, 876:85 (13pp), 2019 May 1

C. Wong et al., arXiv:1907.04869v2 [astro-ph.CO] 6 Nov 2019

Wendy L. Freedman et al., The Astrophysical Journal, 882:34 (29pp), 2019 September 1

Wendy L. Freedman & al., The Astrophysical Journal, 891:57(14pp), 2020 March 1

J. Fleuret, Astrophys. et Space Sci., 350-2, 769 (2014).

J. Fleuret, Astrophys. et Space Sci., 357, 68 (2015)

J. Hu and Y. Liu, Physics Essays, 32, 1 (2019).

J.Fleuret, Physics Essays, 32, 2 (2019).

YA Baurov & al., Bulletin of the Russian Academy of Sciences: Physics. 2015;79(7):935–939.

M. R. Wilczynska et al, Mon. Not. R. Astron. Soc.000, 1–14 (2014) Printed 12 October 2015

Smoot G. Astrophys J Lett. 1992;396.

J.Colin et al., *Astronomy & Astrophysics* (2019).

S. Konstantinov, Journ. Advances in Social Sc. And Humanities, JASSH 6 (3), 1148–1153 (2020)

Migkas K et al., Astronomy & Astrophysics. 2020;636:A15. https://dx.doi.org/10.1051/0004-6361/201936602

W. Olbers, Astromisches Jahrbuch für das Jahr 1826, 150,110-121 (1823).

S.L. Jaki, *The paradox of Olbers' paradox*, Herder & Herder, New York (1969)

B. Mandelbrot, *Fractals: Form, Chance and Dimension*, W. H. Freeman (1977).

T. Piran, *On Gravitational Repulsion*, e-print Arxiv.org/abs/gr-qc/9706049v1, 16 june 1997.

A. Benoit-Levy and G. Chardin, A&A, **537**, A78 (2012).

Y-L. Bolotin et al., Arxiv.org/abs/1502.00811v1 [gr-qc], 3 feb. 2015.

G. Chardin and G. Manfredi, Hyperfine Interactions **239**, 45 (2018) ArXiv:1807.11198v1 [gr-qc] (2018).

G. Manfredi et al., ArXiv:1804.03067v2 [gr-qc], 27 Jun 2018.

J. Belletête et M. B. Paranjape, Arxiv.org/abs/1304.1566v, 4 april 2013.

S. Bondarenko, Modern Physics Letters A, Vol. 34, No. 11, 1950084 (2019)

G-J. Ni, Arxiv.org/abs/physics/0308038, 9 aug. 2003.

S. Konstantinov, Journ. Adv. Social. Sc. and Human. JASSH 6 (3), 1148–1153 (2020)

J-P. Petit et G. dAgostini, Arxiv.org/abs/1408.2451v1, 29 may 2014